**Semi-Analytical Solution for Thermo-Poro-Elastic Stresses in a Wellbore Cement Plug and Implications for Cement Properties that Minimize Risk of Failure**

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**Abstract**

Cementing materials used for plugging wellbores are subjected to evolving temperature, stress, and pore pressure conditions during their service lives. The induced pore pressure changes can be problematic, especially in high temperature and high pressure (HTHP) environments and especially in low permeability materials. However, a design goal of most cement plugs is to achieve very low permeability, with the idea that lower permeability leads to better isolation. Here, with aid of a new semi-analytical solution for thermo-poro-elastic (TPE) stresses in a cylindrical cement plug that includes consideration of full coupling between hydraulic and thermal transport models (so-called “porothermoelastic-osmosis-filtration”, or “PTEOF” model), this work shows that lower permeability is not always better. Specifically, the solution shows that materials that are unable to drain excess pore pressure quickly enough compared to the rate at which these pressures build due to thermal changes are more prone to generate regions of internal tensile effective stress and hence are more likely to be damaged. The specific parameter groups associated with this “permeability penalty” are obtained through a combination of dimensional and pairwise bivariate analysis. These approaches give rise to two dimensionless groups of parameters that are mainly associated with propensity to generate TPE tensile effective stresses. The parametric space defined by these two groups is shown to have three distinct regions based on the probability of generating tensile effective stresses in a plug with a given set of material properties. By shifting the focus of material design from achieving the lower possible permeability to instead achieving the lowest permeability that will not incur increase likelihood of failure due to pore pressure buildup, this work provides a new design concept for wellbore cement. Furthermore, this work highlights for the first time the important role of specific heat of the cement in preventing pore pressure buildup, thereby showing a new way forward for cement design to increase this quantity.

Keywords: Wellbore Cementing; Plug and Abandonment (P&A); High-temperature High pressure (HTHP); Thermo-poro-elastic; Thermal Osmosis; Thermal filtration.

1. Introduction

Properly designed and executed wellbore cementing and plugging operations are important for various geotechnical applications. These include radioactive waste disposal, deep-well plug and abandonment (P&A), drilling and completion in unconventional reservoirs, Enhanced Geothermal System (EGS), and carbon capture utilization and storage (CCUS) (Gruber et al., 2021; Hargis et al., 2021; Koťátková et al., 2017; Olson et al., 2015; Vrålstad et al., 2019). Despite the advancements in cementing materials over the last several decades, the quality of cementing is still often associated with some deficiencies, mainly due to the harsh environments where cement is placed (Ahmed et al., 2020; Allahvirdizadeh, 2020; Kiran et al., 2017). Taking wellbore P&A as an example, over the years and across companies, the upper range of the reservoirs’ pressure and temperature have been pushing up to 275 MPa and 315 Celsius (DeBruijn et al., 2008; Khalifeh et al., 2020). However, the cement is originally designed for low-temperature and low-pressure conditions. Under harsh wellbore conditions, its stability over an extended period of time is unknown. To mitigate this problem, extensive research has been focusing on reinforcing the cement by inclusion of various additives aiming to provide better mechanical and hydraulic properties, with the goal of maintaining the system integrity under the extreme conditions (Cai et al., 2022; Ge et al., 2018; Katende et al., 2020; Krakowiak et al., 2018; Massion et al., 2021; Massion et al., 2022; Qin et al., 2021; Samarakoon et al., 2022).

While much effort has been focused on development of materials and additives, the identification of what comprises “better” mechanical and hydraulic properties of cement are still unclear. This is especially true for high temperature and high pressure (HTHP) environments.

Wellbore cement can be classified as cementitious saturated porous material with permeability ranges from milli- to nano-Darcies (Banthia et al., 1989; Goto et al., 1981; Meng et al., 2021; Picandet et al., 2011). Such a material can be heavily influenced by thermo-hydraulic-mechanical (THM) coupling in the pore space, especially when it is experiencing large temperature and pressure variations. Within the permeability range mentioned above, a very large pore pressure could be induced by the THM coupling, which will also lead to changing of the effective stress and increase the possibility failures (Ghabezloo et al., 2010). While THM coupling phenomena in porous media has been studied extensively, the wellbore-related applications have been mainly focusing on wellbore stability during drilling and fluid injection into borehole (Gao et al., 2017; Song et al., 2019; Tao et al., 2010; Zhou et al., 2009). The THM coupling effect in the cementing designs and performance has rarely been considered and fully investigated under the HTHP conditions. This knowledge gap means that conditions that will generate stresses that are able to damage the material are not well understood, and therefore it has potentially serious consequences.

To include these mutual interactions between thermal, hydraulic, and mechanical systems in the non-isothermal conditions, Biot (Biot, 1977) extended the traditional theory of poromechanics (Biot, 1941) to include the uncoupled thermal effects by incorporating the thermo-molecular diffusion and dynamic forces using a variational Lagrangian thermodynamics approach. Later on, the thermal diffusion process was coupled in solid and fluid deformation by Derski (1979), as well as others (Bear et al., 1981; Kurashige, 1989; Smith et al., 1993). The abovementioned porothermoelastic formulations include an assumption to neglect the non-linear term associated with connective heat transfer, which is thought to be most appropriate for low permeability materials (Chen et al., 2005; Delaney, 1982; Gomar et al., 2014; Wang et al., 2003). Within the framework of linear porothermoelasticity, past studies have been performed for coupled THM behavior of isotropic porous media, but most of the works are assuming the fluid flux and heat flux are dominated by the pore pressure gradient and thermal gradient, respectively (Ghassemi et al., 2002; Ghassemi et al., 2009; Valov et al., 2022). That is to say, the thermo-osmosis denoted by (fluid flux generated by thermal gradient) and mechano-caloric effects denoted by (heat flux generated by pore pressure gradient) are neglected in the transport equations

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Here is the fluid flux,  is the heat flux, *p* is the pore pressure, and *T* is the temperature field. Note that the mechano-caloric coefficient is also known thermal filtration coefficient (Cheng, 2016).

Although thermo-osmosis and mechano-caloric effects are often neglected (if for no other reason, this assumption greatly simplifies solution methods), for porous material with low permeability, these two effects can play important roles (Gonçalvès et al., 2010; Roshan et al., 2015; Trémosa et al., 2010). For example, Carnahan (1983) has shown that the thermo-osmotic flow through kaolinite can be two orders of magnitude higher than Darcy’s flow (that is, the fluid flux driven by the pressure gradient term in Eq. (1)) near a nuclear waste repository. Thus, when designing the cementing under the HTHP conditions, both the thermo-osmosis and mechano-caloric effects should be taken into consideration and should not be dismissed at the outset of the solution. To the best of our knowledge, under the HTHP conditions, their influences on the cement integrity are still unclear.

By introducing a fully-coupled porothermoelastic model, which incorporates both of the thermo-osmosis and the mechano-caloric (thermal filtration) effects, dubbed here as “porothermoelastic-osmosis-filtration” (PTEOF), the present work uses a cylindrical geometry and boundary conditions inspired by cement plugs for P&A as an example to highlight the cementing challenges that are associated with HTHP conditions. The motivation of creating the PTEOF model is to have a comprehensive understanding of the cement behavior under HTHP conditions and to build up a general framework and solutions for future cementing studies. Specifically, we derive a semi-analytical solution (analytical up to a numerical inversion of a Laplace transport) that draws inspiration from the method of Sarout et al. (2011) and therefore leverages the mathematical similarity between PTEOF and linear chemo-poroelasticity. After presenting the governing equations, solution method, and examples of the behaviors predicted by the model, the key parameters associated with preventing tensile effective stresses from developing in a cement plug are identified. These are identified through a combination of dimensional analysis and pairwise bivariate analysis, leading to dimensionless groups that define a parametric space with regions that are “safe” and regions that are associated with material parameter combinations more likely to sustain damage. The work concludes with a discussion of implications for design of cement materials that do not just pursue the lowest possible permeability, but rather pursue a combination of material properties that will provide the necessary isolation without incurring elevated risk of damage from PTEOF phenomena under HTHP conditions.

2. Governing Equations

The formulation begins with classical tensorial strain-stress constitutive relation that expatiates upon the coupled thermo-hydro-mechanical behaviors of fluid saturated porous medium under the plain-strain conditions could be rewritten as (Cheng, 2016)

, (3)

where  and  is stress tensor components matrix and strain tensor components matrix, respectively; is the volumetric strain (the trace of the strain tensor); is the second rank identity tensor;  is pore pressure change from virgin pore pressure and  is temperature change from the reference temperature ;  is shear modulus and  is Poisson’s ratio;  is the Biot effective stress coefficient and  is the thermoelastic effective stress coefficient. Note that, following the sign convention in Detournay et al. (1988), positive stress is considered to be tensile within the present work.

Furthermore, based on the fundamental work of Cheng (2016) and Wang (2017), within the framework of linear thermoporoelasticity, the coupled volumetric response relations are

, where , (4)

Note that one of these three equations can be obtained from contraction on Eq. (3). In Eq. (4),  and  are volumetric strain and stress, respectively, given as the traces of their respective tensors;  is the variation of fluid content per unit volume;  is entropy density. The material constants include the drained bulk modulus tensor, Biot effective stress coefficient, Skempton pore pressure coefficient , coefficient of volumetric thermal expansion of porous media frame  , coefficient of volumetric thermal expansion of variations in fluid content in the solid-fluid system  (Cheng, 2016), and  represents the specific heat of the porous medium at the reference temperature.

Before moving on to the solution method, it is intuitive to reflect that from Eq. (4), deformation of the solid frame is caused by changes in stress, pore pressure, and/or temperature. The fluid phase in the porous medium is not only deforming with the solid frame, but at the same time, driven by pore pressure gradient and thermal forces, causing the pore fluid to be entering or leaving the solid frame of unit volume. Similarly, the stress and temperature change will cause the change of the entropy of the porous system based on the generalized-energy relation. The entropy density is therefore a function of volumetric strain of the solid frame, fluid content, and the change of temperature. Thus, the constitutive equations relate and couple volumetric strain, fluid content, and energy variables  with total stress, pore stress, and temperature variables with the material constants. The notation that used within this work is summarized in Table 1.

Table 1. Notation

|  |  |  |
| --- | --- | --- |
| Symbol | Definition | Unit |
|  | Biot effective stress coefficient | --- |
|  | Drained thermoelastic effective stress coefficient | N⋅m-2⋅K-1 |
|  | Skempton pore pressure coefficient | --- |
|  | Coefficient of volumetric thermal expansion of fluid | K-1 |
|  | Coefficient of volumetric thermal expansion for variation in fluid content | K-1 |
|  | Drained coefficient of volumetric thermal expansion of porous medium frame | K-1 |
|  | Specific heat of the porous medium at reference temperature | J⋅m−3⋅K−1 |
|  | Fluid flux | m⋅s−1 |
|  | Heat flux | J⋅m−2⋅s−1 |
|  | Permeability | m2 |
|  | Thermal conductivity | W⋅K−1⋅m−1 |
|  | Mechano-caloric coefficient (Thermal filtration) | m2⋅s−1 |
|  | Thermo-osmosis coefficient | m2⋅s−1⋅K−1 |
|  | Poisson’s ratio | --- |
|  | Bulk modulus | N⋅m-2 |
|  | Shear modulus | N⋅m-2 |
|  | Thermal Diffusivity | m2/s |
|  | Fluid Diffusivity | m2/s |

Next, based on quasi-static equilibrium, the divergence of the stress tensor is taken to be zero, , that is

, (5)

where volumetric stress  is the trace of the stress tensor. Furthermore, if the displacement is , then the classical small strain assumption is adopted whereby the strain-displacement relations are

 . (6)

Next, we consider the fluid in the pore spaces to be incompressible so that the divergence of the fluid flux () is directly balanced by rate of change of the variation of fluid content (), hence (Cheng, 2016)

. (7)

A similar conservation law assumes the heat flux only through conduction and relates the divergence of the heat flux () to the rate of change of the entropy density () according to (Cheng, 2016)

. (8)

By substituting both transport laws (Eq. (1) and Eq. (2)) into the conservation laws, the fully coupled diffusion equations are obtained as

, (9)

and

. (10)

In these equations, the balancing of the rate of change of fluid content and entropy density with the Laplacian of the fluid pressure and temperature, respectively, comprise the classical uncoupled diffusion equations. However, these equations also have off-diagonal terms that relate rate of change of fluid content to the Laplacian of the temperature field, as well as the rate of change of entropy density to the Laplacian of the fluid pressure. These impacts are known as the thermal osmosis effect and thermal filtration effect, respectively. The model fully coupling all of these terms will henceforth be referred as the porothermoelastic-osmosis-filtration (PTEOF) model. While it may indeed be valid at times to neglect thermal osmosis and/or thermal filtration, here we retain the full coupling in order to elucidate conditions in which they may have an important impact.

With the addition of boundary and initial conditions (to be discussed later), Eqs. (1-10) comprise a complete model, sufficient to solve for all unknown quantities.

3. PTEOF Solution for Constrained Cylinder

3.1 Problem description and boundary conditions

Consider a thermoporoelastic cylinder of radius R and length 2L (Figure 1a). Both of its ends are jacketed which is fixed and impermeable for fluid and thermal isolation from heat. The radial boundary conditions are unjacketed and can have fluid and heat exchange at the boundary. This constrained cylinder geometry is inspired by the geometry and boundary conditions of the primary plug in P&A (Figure 1a) where the length of the primary plug is usually 50 to 100 times larger than its diameter that often ranges from 5 to 20 inches and its loadings mainly come from the high temperature, and pore pressure from the formation, and the far-field in-situ stress. Thus, it is appropriate to apply the generalized plane-strain assumption where the pore pressure and thermal diffusion only appear in the isotropic plane that is perpendicular to the length axis of the plug which is fully saturated.

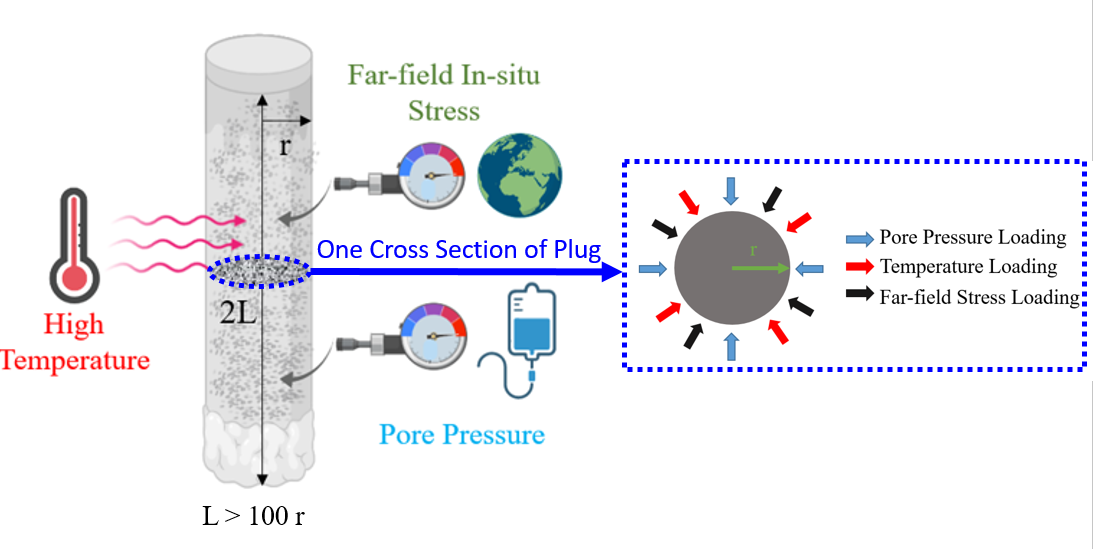


Figure 1(a) Sketch showing a primary plug in P&A and its boundary conditions; (b) A zoom in sketch showing a cross-section of the plug in plane-strain conditions and its boundary conditions.

In line with the loading decomposition scheme proposed by Detournay et al. (1988) in the context of poroelasticity, the PTEOF model can be decomposed into three sub-loading cases to simplify the analysis. These are: 1) pore pressure loading, 2) temperature loading, and 3) isotropic far-field stress loading. Thus, the boundary conditions at the outside surface of the primary cement plug for each of the loading modes can be written as follows (the subscript f denotes formation, subscript c denotes cement)

* Mode 1: ,, and , at  ;
* Mode 2: ,, and , at ;
* Mode 3: ,, and , at ;

where  is the normal stress in the radial direction, and  is the Heaviside step function. Since the PTEOF model is linear, the principle of superposition will be used as final step to obtain the final solution by combining the individual loading modes. Note that, in general, there is a fourth mode of loading corresponding to deviatoric far field stress, which is neglected here but can be useful as a future investigation.

3.2 Solution to the fully coupled diffusion equations

The solution method starts with obtaining a general solution of the fully coupled diffusion equations (Eq. (9) and Eq. (10)). Note that the plane-strain assumption will lead to fluid and thermal transport directed only along the radial direction in the cylindrical coordinates system. Then, the first step is to eliminate  and  in the diffusion equations by substituting the constitutive equations (Eq. (4)). This results in a new form of coupled diffusion equations

, (12)

, (13)

where , and .

Next, by combining Eq. (3), Eq. (4) and Eq. (5), and taking the body force to zero, an extended form of the classical Navier equation is obtained as

. (14)

Drawing on the approach of Sarout et al. (2011), we can then use the irrotational field assumptions to simplify the process of solving the Navier equations. When displacement field is irrotational, i.e.  is the gradient of a scaler, one can integrate Eq. (14) to obtain

, (15)

where  and .

The functionresults from the integration and it therefore does not depend on the spatial coordinates. In fact, for many infinite or semi-infinite domain problems, conditions exist to require quantities decay to zero in the far field, thus forcing to *f(t)* to be zero (Detournay et al., 1993). But, for the present finite domain problem,will not be zero but instead is a part of the solution. Next, by substituting Eq. (15) into Eq. (12) and Eq. (13), the volumetric strain  can be eliminated thus leaving the coupled diffusion equations

, (16)

where:

, (17)

, (18)

, (19)

, (20)

, (21)

, (22)

. (23)

The diffusion equations in Eq. (16) are coupled in terms of  and , which can be transformed into uncoupled equations in terms of  and by using the eigen decomposition approach developed by Sarout et al. (2011). This methodology starts by obtaining two eigenvalues  and  of the matrix. Then, the eigen decomposition theorem (Weisstein, 2002) allows us to define a transition matrix , which is composed of eigenvalues and eigenvectors of . The temperature and pore pressure therefore be changed into new variables by

 , (24)

where

,

.

Substituting this change of variables into Eq. (16) leads to an uncoupled system of diffusion equations and this process can be developed based on the eigendecomposition theorem

, (25)

where . Then, by applying the Laplace transform to the eigenfunction

, (26)

where , one obtains an uncoupled system of ordinary differential equations (ODEs) in terms of the variables , where  is function of the coordinators in cylindrical system ρ, and the Laplace parameter s and the ’s eigenvalue . These ODEs are

. (27)

 are zeroth-order modified Bessel equations, which have the general solution

, (28)

Here  , and  are unknowns that will be determined based on boundary conditions,  and are the zeroth-order modified Bessel functions of the first and second kind, respectively.

The next step is to invoke the symmetry conditions of the problem under consideration, which ensure that =0. Thus the pore pressure and temperature profile in the Laplace domain are

, (29) . (30)

The radial stress under the plane-strain conditions (Eq. 3) consists volumetric strain () and radial strain (). Both of them can be obtained through the radial displacement , which is integrated by the Eq. (15). Thus, the radial stress can be expressed in Laplace domain

, (31)

Where the trace of strain tensor in Laplace domain is

 , (32)

and radial strain is

. (33)

Up to now, there are three unknowns , , and  in the system. The first two unknowns  and come from the coefficient of the zeroth-order modified Bessel functions of the first kind. The third unknown, , is coming from the spatially uniform function from Eq. (27). In the process of deriving the above expressions for the pore pressure, temperature, and radial stress, we can create three equations for these three unknowns

 （34）

where

,,,,

,

,

,

.

So, when given a value of , the three unknowns can be readily computed, thus the solution can be numerically inverted to the time domain using Stehfest’s method (Stehfest, 1970), which has been shown to be efficient in poroelastic problems, and its details are given in Appendix A.

4. Behavior of the Model

This section will investigate different behaviors of the model and an example case is selected in order to have a further illustration. Based on the information provided by the literatures (Addis, 1997; Santarelli et al., 1998; Snee et al., 2018; Zoback et al., 2003), this example involves all three loading modes which are determined by the boundary conditions, specifically:

* Mode 1: a difference between the formation and cement plug is Pa;
* Mode 2: a difference between the formation and cement is taken as  Kelvin;
* Mode 3: a change of far-field isotropic stress is taken as Pa.

The details of the initial conditions will be introduced later in the section 4.3 where the effective stress is discussed. The above mentioned literatures also inspire the other parameters to be taken as

 (35)

4.1 Pore pressure responses and thermal osmosis effect

The change of pore pressure () in response to different loadings is a key component in poromechanical analysis. It is therefore of interest to firstly plot the evolution of pore pressure under three different loading modes, given the boundary conditions that are defined in the last section, while the influence of the factors of interest on the pore pressure will be analyzed and discussed later.

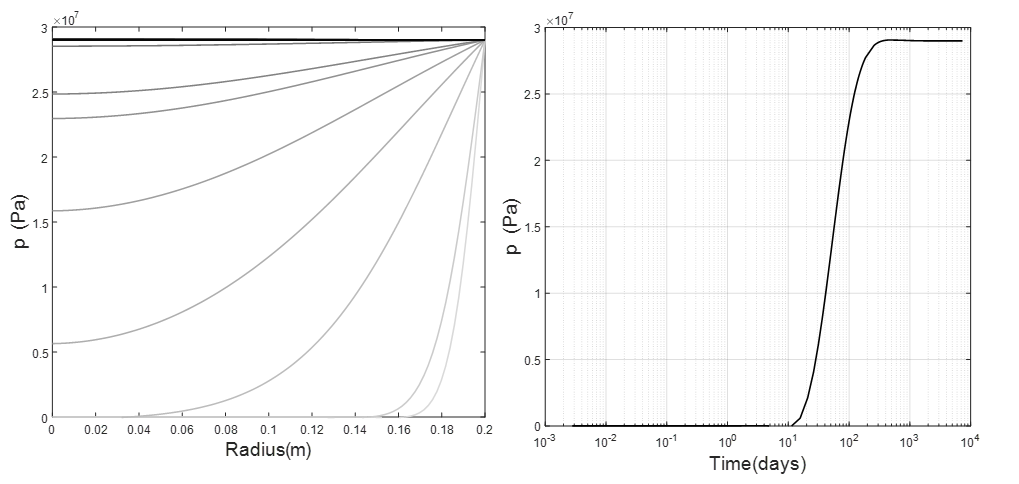


Figure 2. (a) Pore pressure change () due to mode 1 loading, where the curves grade from gray to black as time increases; (b) Evolution of pore pressure change () induced by mode 1 at center of the cylinder ().

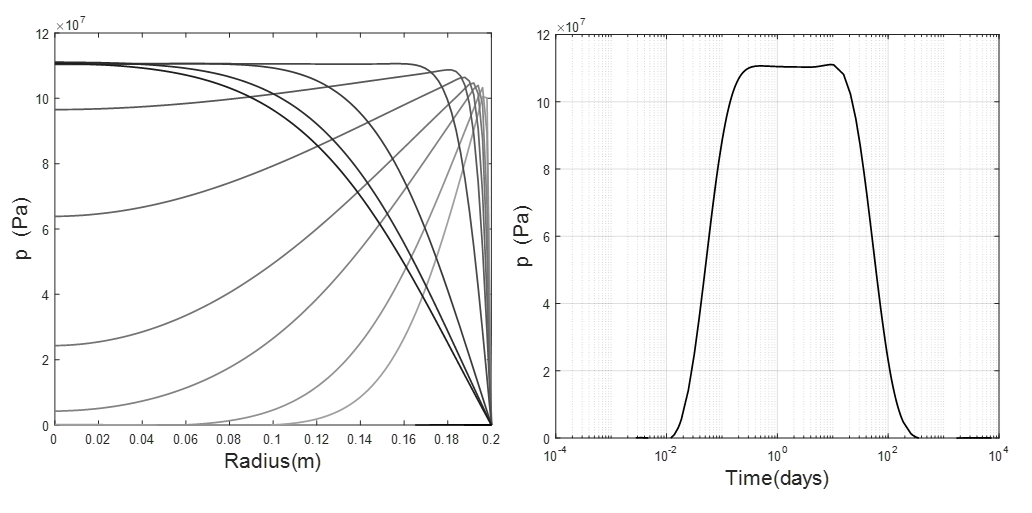


Figure 3. (a) Pore pressure change () due to mode 2 loading, where the curves grade from gray to black as time increases; (b) Evolution of pore pressure change () induced by mode 2 at center of the cylinder ().

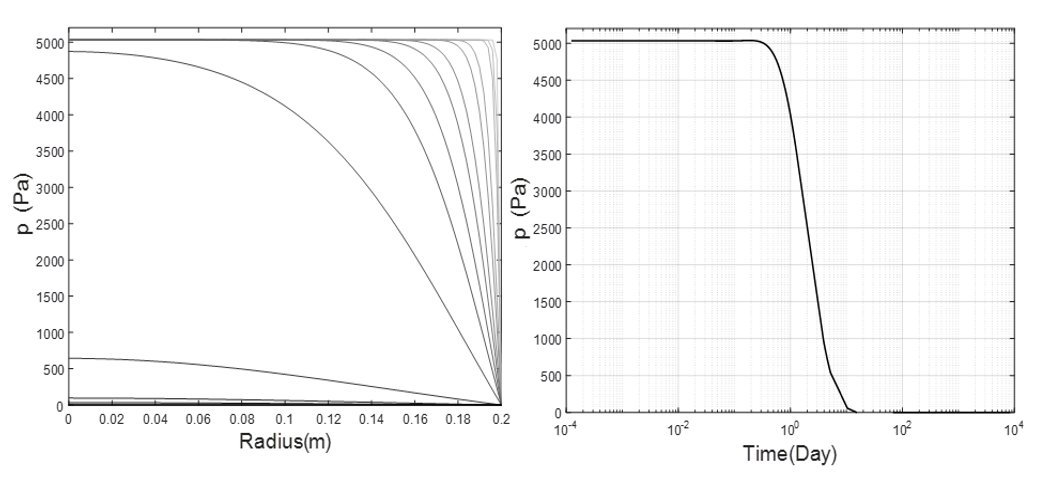


Figure 4. (a) Pore pressure change () due to mode 3 loading, where the curves grade from gray to black as time increases; (b) Evolution of pore pressure change () induced by mode 3 at center of the cylinder ().

Figures 2a and 3a illustrate the pore pressure change () profile along the radius in response to mode 1 loading (pore pressure loading) and mode 2 loading (temperature loading), respectively. Under the mode 1 loading, the pore pressure evolution follows in the manner of a classical diffusion process. The pore pressure near the surface (r=0.2m) is instantly raised to the level of the pore pressure loading, and then the pore pressure gradually diffuses towards the center (r=0).

In contrast to gradual and smooth diffusion process from external pore pressure loading, the pore pressure induced by mode 2 temperature loading firstly peaks just inside the boundary. The pressure change at the boundary in this case is held at zero due to the boundary conditions. Once the initial pressure peak develops, it initially decreases to zero at the inner core, resulting in a pressure peak. This pore pressure peak results in fluid flux both toward the boundary and toward the center. However, because fluid diffusion is slow compared to thermal diffusion, the pressure does not appreciably dissipate. Instead, as the temperature front moves to the core, the pore pressure increases along with it. Eventually the pressure becomes nearly uniform through the central region, while it continues to have a gradient toward the boundary. Hence, the pressure is being alleviated by fluid diffusion across the outer boundary. But, again because of the slow fluid diffusion due to low hydraulic conductivity (in comparison to the thermal conductivity), for some time there persists only a small layer at the boundary where the pressure is able to drain. Eventually, long after the entire specimen is entirely heated, the entire pore pressure dissipates and returns to zero everywhere.

The time evolution of the pore pressure due to modes 1 and 2 loading is illustrated by the pressure at the center, shown in Figures 2b and 3b. After pressure loading, the pore pressure in the center is shown for this example to begin increasing after about 1 month, reaching its peak, steady-state value after about 1 year. Hence, any pore pressure change that takes place over a time that is considerably smaller than this characteristic time of evolution can be approximated by the instantaneous change considered in this solution. The pore pressure due to thermal loading progresses somewhat more quickly, owing to the higher thermal diffusivity. After the thermal loading is induced, the pressure at the center begins to rise very soon thereafter, within less than 1 day. The induced pressure pulse in this example persists for a few months.

In contrast to the previous two modes, the pore pressure change () that is induced by mode 3 (isotropic far-field stress) is shown in Figure 4a. Here the pore pressure immediately arrives at its highest value except in a small layer near the boundary where the pore pressure reduces to zero due to the boundary conditions. This initial pore pressure field gradually declines to zero everywhere due to fluid diffusion. The entire process for this example takes place over the course of about 1 month (Figure 4b). It should be noted that the magnitude of the pore pressure induced by mode 3 is much smaller compared to the pore pressure induced by mode 1 and mode 2. Even at the highest level of pore pressure, which is right after the loading is applied, the ratio of mode 3 induced pore pressure over mode 1 and mode 2 induced pore pressure is around 0.01%-0.02%. This is also in line with the findings from the classical poroelastic model (Detournay et al. (1988).

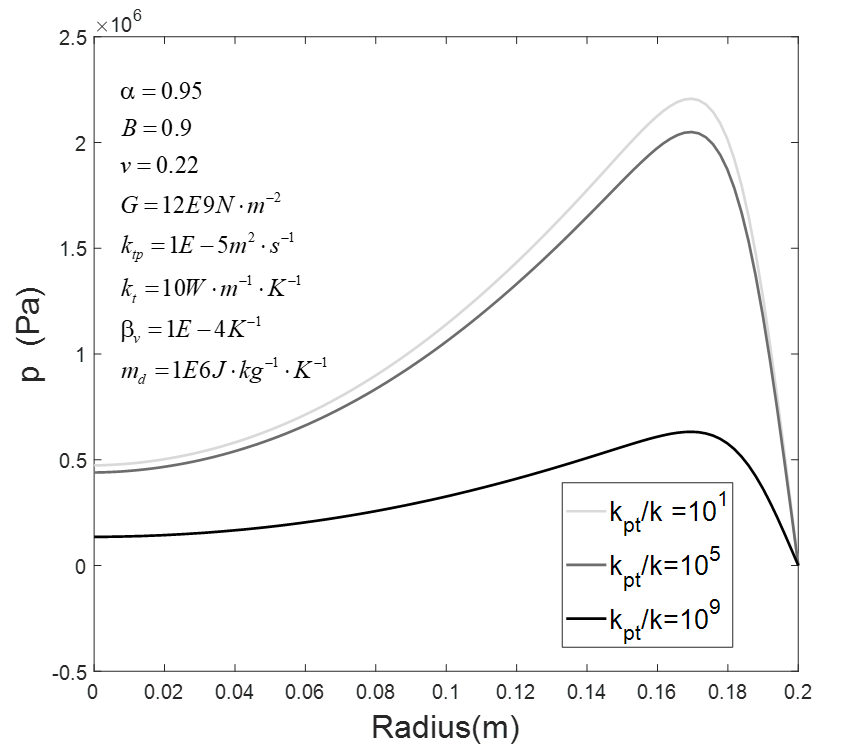


Figure 5. Various ratios of (thermo-osmosis’s influence) impacting the mode 2 induced pore pressure shown at (days).

4.2 Thermo-osmosis and thermo-filtration effects

So far, all results have been presented with a single combination of parameters governing thermos-osmosis and thermo-filtration. To examine these effects, we will present the internal pressure profile at *t*=10 days after thermal (mode 2) loading. Three different ratios (1, 105, 109) of  are selected to show the different developments of the temperature induced pore pressure (Figure 5), noting that =1 corresponds to the case presented in Fig. 3. All three cases show a peak pore pressure near the surface region; however, the magnitude of the peak induced pore pressure has considerable reduction when the thermo-osmotic coefficient is significantly larger than the hydraulic conductivity (i.e., =109). While it is not clear if this ratio is realistic (it might be, but there has been little study), it is clear that thermos-osmosis does have potential to contribute a reduction in the induced pore pressure from temperature loading.

The role of thermo-filtration is made most apparent by observing the temperature profiles resulting from the mode 2 (temperature) loading. A base case is shown in Figure 6. As the time proceeds, temperature is increasing monotonically from surface towards the inner core of the plug, and finally arrives at equilibrium, which is the equilibrium. Three different ratios of thermal conductivity over the thermal filtration coefficients are selected to show the impact of the thermal filtration effect on the temperature profile as it is changed by the pore pressure gradient from mode 1 loading. As shown in Figure 7, among the three selected ratios, the maximum temperature difference induced by the pore pressure gradient is within 1 Kelvin. Furthermore, when the thermal conductivity is significantly larger than the thermal filtration coefficient (=108), the induced temperature differences are negligible under the current model settings and inputs.

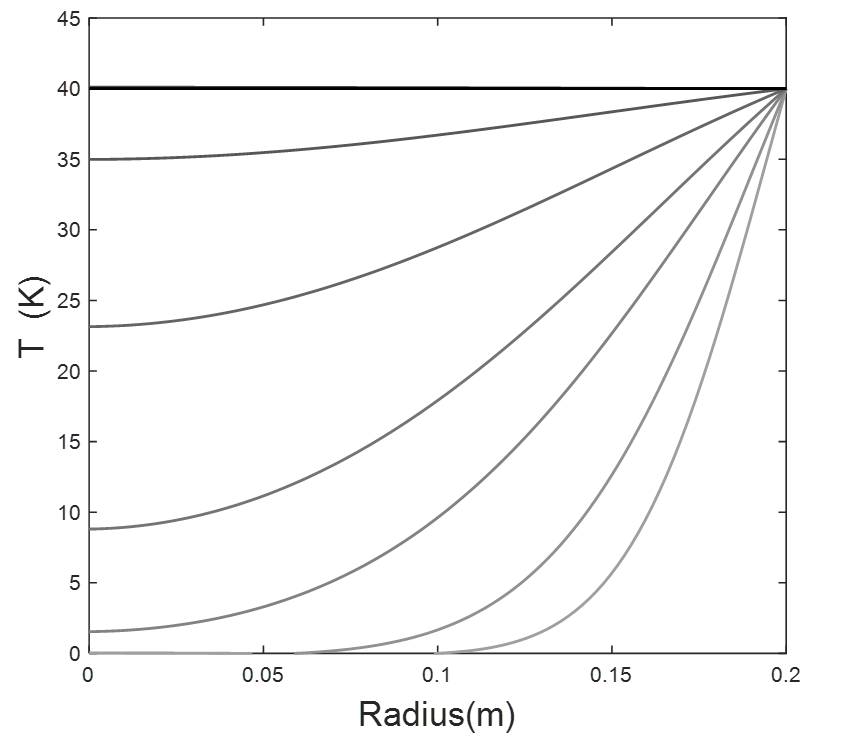


Figure 6. The development of temperature change () under loading mode 2, where the curves grade from gray to black as time increases.

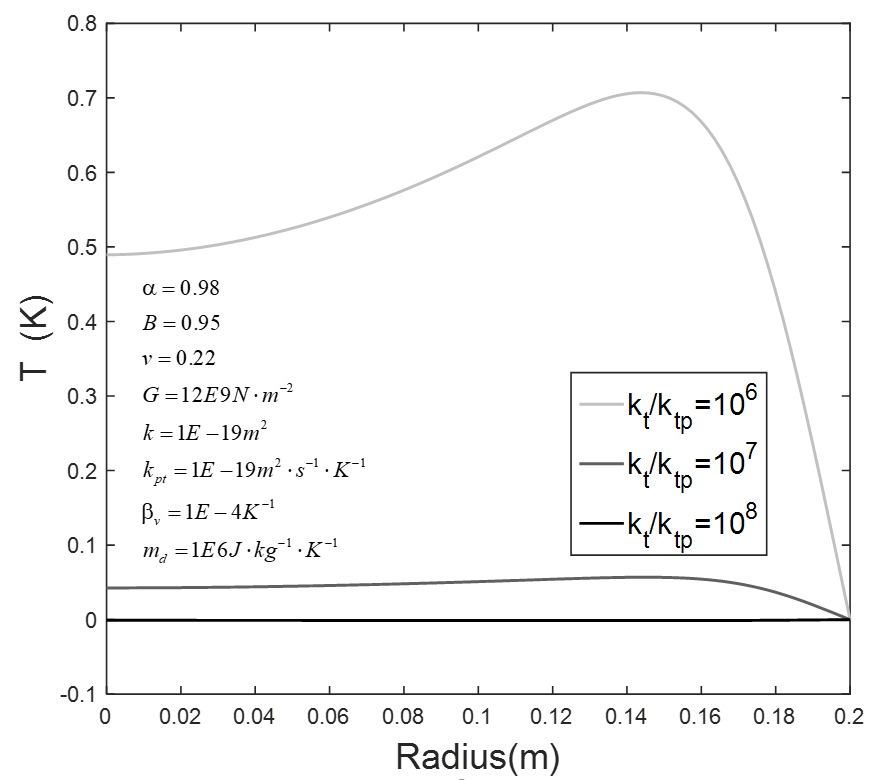


Figure 7. Various ratios of (thermal filtration’s influence) impacting the temperature change () due to loading mode 1.

4.3 Effective Stress

There is a vast literature pointing to the impact of the so-called effective stress on behavior of porous media, where effective stress is defined as the difference between the total stress and the pore pressure (Biot, 1941; Terzaghi, 1925). Its significance lies in that it quantifies the total stress carried by the solid skeleton. Hence, when it becomes tensile, it indicates that the skeleton is subjected to tension. In materials like cement, rock, and soil, the tensile strength is low and so generation of tension indicates risk of tensile failure. Based on the information provided by the literatures (Addis, 1997; Santarelli et al., 1998), the initial conditions of this example are set up as follows: the initial pore pressure of the cement plug is set up Pa; the initial temperature in cement is initially 313 Kelvin; the initial far-field isotropic stress is setup as Pa. The boundary conditions are still remained the same as what stated at the beginning of this section. Here, the effective radial stress  is obtained as the summation of the initial effective stress () and superposition of the effective stress  from three loading modes as

, (36)

where ,, and .

The result is plotted in Fig. 8, recalling that tension is positive. It is observed that after the cement is placed under the pore pressure, temperature, and stress loadings, the induced pore pressure will increase and will therefore reduce the radial effective stress near the boundary. The results show that it can create a region where effective stress is tensile. However, outside of this region and at early times, the rest of the material will still be subjected to compressive effective stresses. Furthermore, because the pore pressure diffusion process is very slow compared to thermal diffusion, the pore pressure continues to increase through to the center and eventually a tensile region will be created throughout the central region. Finally, at a later time when induced pore pressure is fully dissipated, the radial effective stress will return to be compressive again. This demonstration of the whole diffusion process indicates that cementing under the HTHP has potential to induce a high value of pore pressure, which will lower the effective stress and can even generate a tensile region. This coupled behavior can therefore lead to cracking of the cement, jeopardizing the integrity of the cementing system with the potential to trigger unwanted consequences.

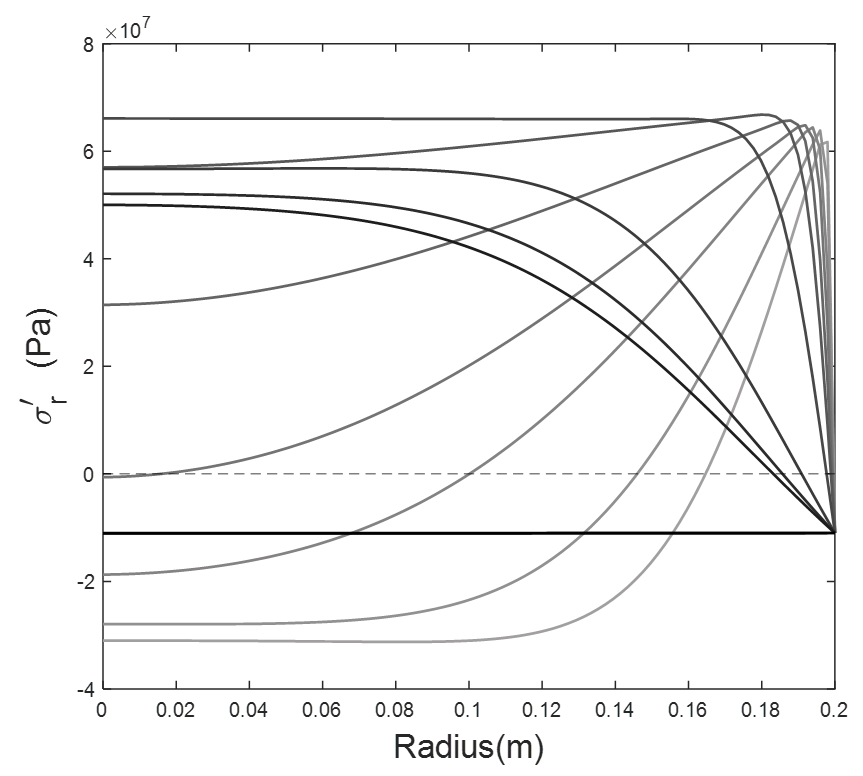


Figure 8. Effective radial stress of cement, with tension positive, where the curves grade from gray to black as time increases (boundary conditions and input values of each parameter are stated at Eq. 35).

5. Pairwise Bivariate Analysis and Dimensional Analysis

The system behaviors discussed so far depend on the different values of input parameters and the mutual interactions of the physical processes they determine. If some of these parameters take on different values, the results can be much different. Thus, it is important to investigate the potential prevalence of the high pore pressure generation and its associated generation of tensile effective stress. The analysis starts with assigning values at random to each parameter within a certain range, as summarized in Table 2. Once these variables have been given values according to this Monte Carlo approach, they are substituted into the semi-analytical solution and a new solution is thus computed. For each solution, the most tensile effective radial stress at any location and any time is extracted from the data and then used to classify the case as “tensile” or “compressive”. The details of the above-mentioned procedures are summarized in the flowchart in Figure 9.

Table 2. Lower boundary and upper boundary for Monte Carlo Sampling

|  |  |  |  |
| --- | --- | --- | --- |
|  | Lower Boundary | Upper Boundary | Unit |
|  | 0.5 | 0.95 | --- |
|  | 0.5 | 0.98 | --- |
|  | 1E-5 | 1E-3 | K-1 |
|  | 12E9 | 24E9 | N⋅m-2 |
|  | 0.22 | 0.32 | --- |
|  | 1E-21 | 1E-16 | m2 |
|  | 1E0 | 1E2 | W⋅K−1⋅m−1 |
|  | 1E-6 | 1E-9 | m2⋅s−1 |
|  | 1E-22 | 1E-19 | m2⋅s−1⋅K−1 |
|  | 1E5 | 1E7 | J⋅m−3⋅K−1 |

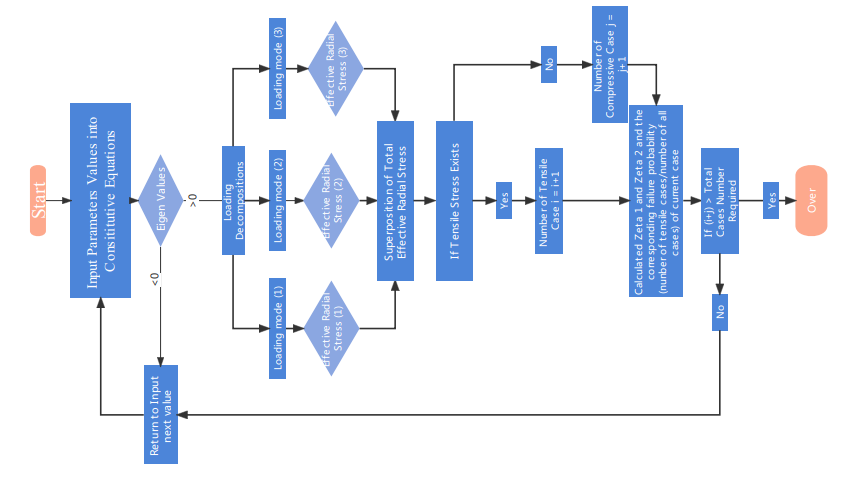


Figure 9. The algorithm that is used to construct the parametric studies.

5.1 Pairwise Bivariate Analysis for Each Variable

Following the procedures detailed in the flowchart described in Figure 9, a total of 3000 cases are calculated and then categorized into two groups: tensile (1914 cases) and compressive (1217 cases). Next, the relationship between each variable and the outcome of “tensile” is examined by the pairwise bivariate distributions that are shown in Figure 10. The non-diagonal elements are scatter plots which display the correlation between two variables and give insight on the distribution features of these variables. The matrix of the results is symmetric about its diagonal. The diagonal elements are univariate distribution plots which are drawn to show the probability density function of each variable. Based on the information provided by the diagonal components, it can be found that in the events of a tensile case, the specific heat and permeability are more concentrated at their lower range, and the thermal conductivity and the thermal expansion coefficient differences are more concentrated at their higher range, whereas the rest of the parameters are almost evenly distributed along their whole range. This indicates a general trend that, within the framework of PTEOF, at higher values of  and lower values of  and  will increase the propensity for generating tensile radial effective stress increases.

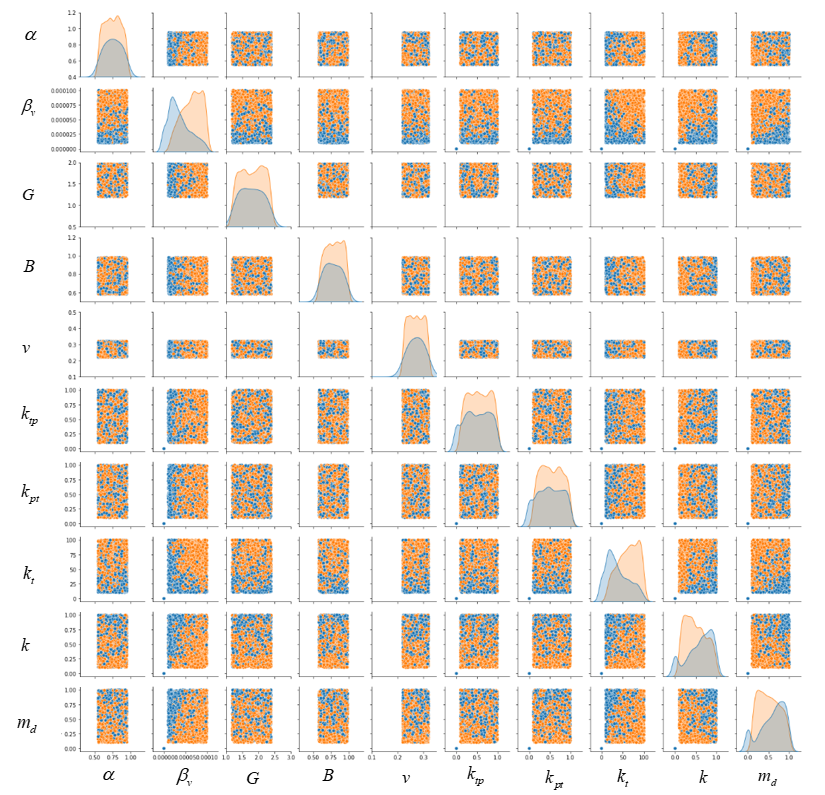


Figure 10. Pairwise bivariate distributions for eleven variables. The blue distribution on diagonals indicate distribution of the parameters for comparative cases, while orange shows distribution for tensile. Difference between these distributions indicate how the parameters impact on the “tensile” versus “compressive” outcomes.

The impact of permeability on tendency to generate tension is a point that bears further discussion. One common cement design principle is that lower hydraulic conductivity is synonymous with a better cement barrier. This cement designing philosophy is possibly effective under many working conditions (i.e., ambient temperature and pressure) for which cement is originally developed. However, when the cement is placed under HTHP conditions, and poromechanics are taken into consideration, our results show that the lower permeability will actually increase the probability of the cement experiencing tensile effective stress, which can be detrimental to the integrity of the cementing system. This is mainly due to the fact that under the framework of porous media, the lower permeability will greatly slow down the pore fluid diffusion process when the pore pressure is rapidly built up within the cement due to the HTHP boundary conditions. Hence, the pursuit of low permeability regardless of the intrinsic porous properties of the cement itself and without guidance from poromechanical models could instead increase likelihood of degradation of the cement and hence reduce its effectiveness at providing mechanical support and zonal isolation. In other words, there is a negative aspect to having permeability that is too low, which we will henceforth call a “permeability penalty”. To avoid the permeability penalty for cementing design under HTHP, perhaps the most favorable solution is to keep the permeability at certain ranges which can achieve the sealing function, but at the same time, allowing the diffusion of pore pressure that is built up by the HTHP conditions and therefore not cause unnecessary damage induced by the excess pore pressure. This permeability-forgiveness design is actually very popular in pavement design of permeable porous systems (PPS) (Scholz et al., 2007) where one or two special drainage layers with relatively higher permeability are designed to reduce the runoff rates during storms while providing a hard surface for the traffic flow. The PPS design has been successfully turned into wide variety of residential, commercial, and industrial applications in the last two decades (Drake et al., 2013). Furthermore, finding a suitable permeability window should in principle be possible for wellbore cementing owing to the vast difference between the length scale associated with drainage of pore pressure to a radial boundary and the length scale associated with fluid diffusion through the length of the barrier. A permeability that is high enough to allow the former while preventing the latter should be attainable.

It also worth mentioning that the specific heat capacity has received little attention in previous research of HTHP cementing design. However, based on our results, it does play a very important role, because a large specific heat capacity will effectively slow down the heat transfer process and hence reduce the tendency to generate thermally-induced pore pressure. This will give the pore-pressure more time to dissipate compared to the rate of its build-up. This observation suggests a new direction for creating cement with high specific heat capacity and low thermal conductivity which would comprise suitable design for HTHP conditions.

5.2 Parameter Groups Governing Effective Stress Evolution

Although the pairwise bivariate distributions shown in Figure 10 gives a full picture of how each parameter will individually influence the system behavior, the mutual interactions of different properties and their group effects cannot be analyzed from it. To solve this problem, classical scaling and dimensional is utilized. Usually these methods are used to reduce a complex physical problem to a simpler version (at least in terms of the number of independent governing parameters) prior to obtaining a quantitative answer while also grasping the effects of various physical phenomena at the same time. With that said, it is not clear from the governing equations alone which grouping(s) of parameters have the most important effects on the propensity to generate tensile effect stress. However, by dimensional analysis and guided by the features that the diagonal elements from the pairwise bivariate distributions (Figure 10) provide, one can propose a dimensionless parameters as

, (37)

 (38)

 (39)

This dimensionless group,, consists of the Biot coefficient , Skempton coefficient , thermal expansion coefficient of fluid  and solid  , as well as fluid diffusivity  and thermal diffusivity , where the last two terms are obtained from the diagonal elements of the coefficient matrix (Eq. 17 and Eq. 20). This proposed mainly contains the hydro-thermal properties of the porous media. It is thus named the Hydro-Thermal Non-Dimensional parameter (HTND).

At the same time, if the variation of fluid content  in Eq.(4) is set as zero (undrained solution), one can eliminate the volumetric stress  in the same equation, and thus obtain as

, where . (40)

If one firstly inverts Eq. 32 into time domain and then inserts Eq. 38 into it, by integration

. (41)

Then by inserting Eq. 39 into Eq. 31 in time domain, and setting up the  as zero given the boundary conditions of the mode 2 loading, the thermal induced pore pressure given a temperature rise is

. (42)

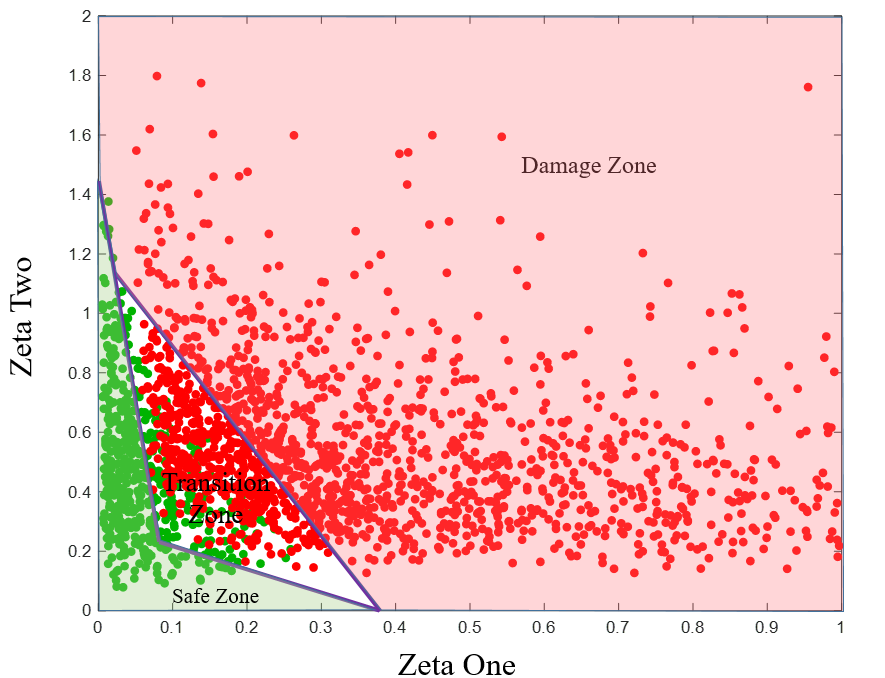
Thus, the second proposed parameter is proposed as

, (43)

which consists of thermal induced (mode 2) pore pressure () and the in-situ stress ().

Given the newly proposed parameters, a new plot can by generated to show the tensile and compressive spatial distribution cases based on and . In Figure 11 (a), the same 3000 cases are replotted where the red dots represent the tensile cases and the green dots represent the compressive cases. It can be clearly seen that the spatial distributions of the tensile and compressive cases in the  vs.  plot are dividing the sampling feasible area into three zones with clear boundaries: safe zone, transition zone and damage zone. Futhermore, in Figure 11(b) where is equally divided into 200 intervals, the probability of generating the tensile cases in each of these intervals is calculated. If just considering as a safety index solely from the plugging material’s perspective, it is shown that in the area where theare smaller than 0.07 is fully occupied by compressive cases; not a single tensile case exists in this region. It indicates no tensile failure would be expected to occur within this range under the framework of the PTEOF model (and for the particular boundary and initial conditions considered in this example). When  increases from 0.07 to 0.3, the probablity of generating tensile cases gradually increases up to 100%. Both tensile and compressive cases could happen in this region, but with a higher value of , the possibility of generating a tensile case will be higher. This area is where the transition zone mostly locates. When is greater than 0.3, no compressive case is exists within this range and it is fully occupied by tensile cases, which means the probality of generting a tensile cases is 100%. It is thus named the damage zone which means when designing the cement under the HTHP conditions, if the resulting is landing greater 0.3, tensile damage will certainly occur in the cement.

Drawing on the analyses and conclusions above, it can be clearly seen that the Shear Modulus and Poisson’s ratio, which are not included in  but traditionally considered as two of the most important mechanical properties, have very limited contribution to the system behavior when in terms of its propensity to generate poromechanical tensile effective stress. So, this work suggests that from at least this one perspective, more attention should be put into the components that keep the value of below 0.07, as to avoid potential for tensile damage. For example, as shown in Figure 12, the Peak 1 is recalled from the Figure 3(b), giving a case for which equals to 1.26. This case is firmly in the “damage zone”. However, if the permeability of the material is increased by two orders of magnitude, the induced pressure is much lower, shown here as Peak 2. Still, the value of  and Figure 11(b) suggests a probability of 92% to generate tensile effective stress. If the specific heat of Peak 2 is further increased by two orders of magnitude, the induced pore pressure will have an even more considerable reduction (Peak 3). The corresponding = 0.03, which is indicated in Figure 11(a) to be in the “safe zone” with negligible probability of generating tensile effective stress.



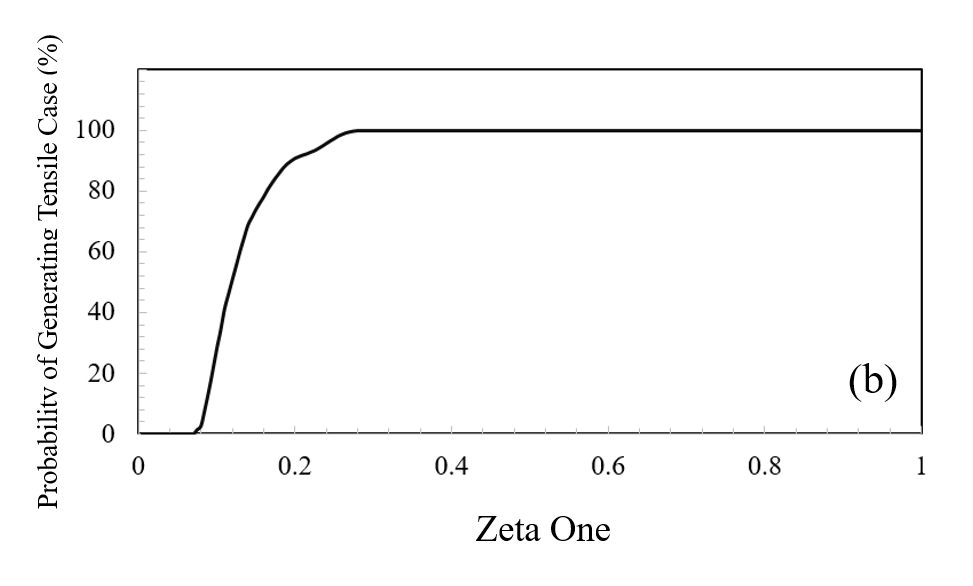


Figure 11. (a) (HTND) and plot with tensile cases (red color) and compressive cases (green color). Note the range of Zeta One can be larger than 1 based on the ranges of the selected parameters. However, the probability of generating the tensile case is always 100% when Zeta One is larger than 1. (b) The probability plot of generating tensile cases on different  (HTND) values.

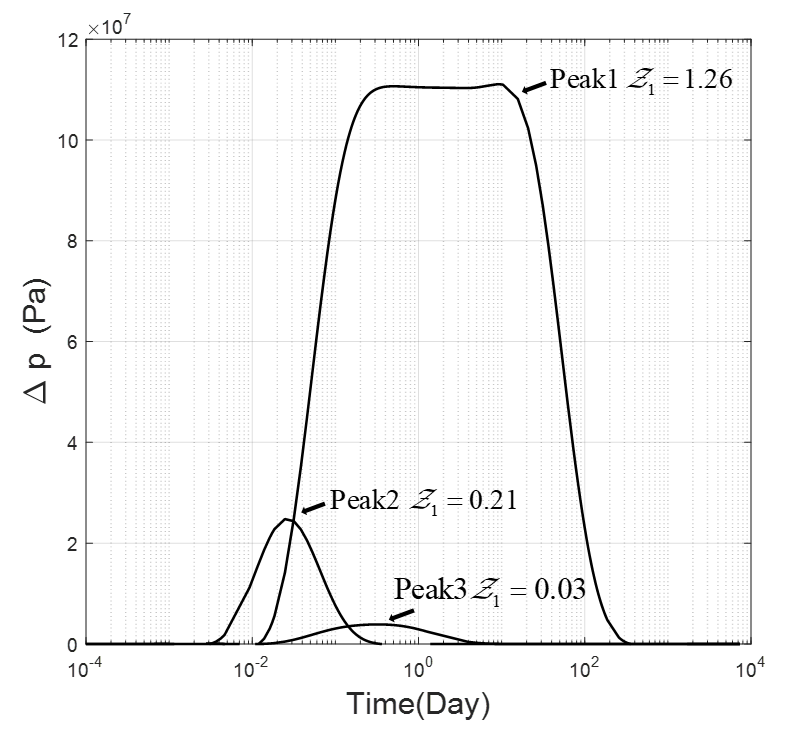


Figure 12. The influence of Zeta 1 on the Mode 2 induced pore pressure. Peak 1 is recalled from Fig. 3 Right; The permeability of Peak 2 is three orders of magnitude smaller than Peak 1, the rest of selected parameters remain the same as Peak 1; The specific heat of Peak 3 is two orders of magnitude smaller than Peak 2, the rest of selected parameters remains the same as Peak 2.

6. Conclusions

Using the primary cementing in P&A as an example, the fully coupled porothermoelastic-osmosis-filtration (PTEOF) model is presented here. It has been demonstrated that under HTHP conditions, the induced pore pressure in low permeability material such as cement can be so large that it can generate tensile effective stresses. This phenomenon will, in general, increase the probability of cement failure. By the pairwise bivariate analysis, the properties that are important for the HTHP cementing are parsed out. Based on these results, two new concepts are proposed in which: 1) permeability targets a desired range that is large enough to facilitate timely dissipation of pore pressure due to radial fluid flow while still giving the necessary zonal isolation by preventing axial flow through the length of the plug, and 2) seeking methods to substantially increase the specific heat of the cement. Furthermore, by scaling analysis and Monte Carlo simulation, three zones (safe, transition, and damage) with clear boundary values are shown in the  and plot. Of these,  is by far the most influential, showing that cement design with higher permeability and specific heat is expected to reduce the likelihood of pore-pressure induced failure as the cement experiences temperature changes during its service life.

A practical implication is that thermal properties, such as thermal conductivity, specific heat capacity, thermal expansion coefficient ratio, deserve more attention in cement design for HTHP conditions. The potential impact of increasing the specific heat capacity is substantial, especially because it allows mitigation of pore pressure buildup without the need to deliberately increase the hydraulic diffusivity (which may not be desirable for other reasons).

These results show the need for mechanical modeling to guide design of cementing materials for HTHP conditions. Common cement improvement practices, such as including different additives into cement to enhance its mechanical properties, will not change the fact that cement is inevitably a porous medium. While the new additive can change the pore spaces into various sizes as small as the nano-scale, without considering the underlying physical principles governing the mechanical behaviors within a poromechanical framework, modifications of properties with on objective in mind could inadvertently increase risk of failure due to another mechanism. On the other hand, guided by fully-coupled modelling, the future of cement design for HTHP conditions can be more effective by pursuing directions that may not be apparent without the use of a poromechanical model.

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Appendix A: Details of Stehfest’s method

For a given function p with Laplace transform, Stehfest’s method (Stehfest, 1970) can be expressed as, taking pore pressure for example:



Where the coefficient are given by



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